

Newtonian gravity as an entropic force: Towards a derivation of G

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Abstract. It has been suggested that the Newtonian gravitational force may emerge as an entropic force from a holographic microscopic theory. In this framework, the possibility is reconsidered that Newton's gravitational coupling constant G can be derived from the fundamental constants of the underlying microscopic theory.

Journal-ref: Class. Quantum Grav. 28 (2011) 125003

Preprint: IPMU10-0098, KA-TP-15-2010, arXiv:1006.2094

PACS numbers: 04.50.-h, 05.70.-a, 06.20.Jr, 04.80.Cc

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1. Introduction

Recently, Verlinde [1] has given a heuristic argument of how space, inertia, and gravity could emerge from a microscopic theory in a holographic approach [2, 3]. Gravity would arise as a type of entropic force. (Related ideas have been presented in, e.g., [4, 5].)

Verlinde's discussion of Newton's law of gravity is particularly elegant, as it directly gives an inverse-square law for the attractive force between two macroscopic point masses M_0 and M_1 . Specifically, the force on the point mass M_0 at position \mathbf{X}_0 due to an effective point mass \widetilde{M}_1 at an effective position $\widetilde{\mathbf{X}}_1$ (the mass \widetilde{M}_1 corresponding to a spherical holographic screen) is given by

$$\mathbf{F}_{0,\text{grav}} = M_0 \mathbf{A}_{0,\text{grav}} = G M_0 \widetilde{M}_1 (\widetilde{\mathbf{X}}_1 - \mathbf{X}_0)/|\widetilde{\mathbf{X}}_1 - \mathbf{X}_0|^3, \quad (1)$$

with $\mathbf{A}_{0,\text{grav}}$ the acceleration of the mass M_0 .

In this article, a previous suggestion [6] is reconsidered that Newton's gravitational constant G can be derived from more fundamental constants of nature, including a new fundamental length l (see also [7, 8] for a classic paper and a recent review). The entropic explanation of Newtonian gravity then gives a new interpretation of an earlier formula [6] for the Newtonian gravitational acceleration originating from a macroscopic point mass. Moreover, having a new fundamental constant l may help in resolving a potential problem of Verlinde's approach regarding the total entropy of a general equipotential screen. Restricting the screen to a black-hole horizon, this entropy can be used to perform a model calculation of G and to get a numerical estimate for l by connecting to the Bekenstein–Hawking entropy [9, 10]. At the end of this article, a few comments are presented on possible experiments to determine this new fundamental constant l , if really existent.

2. Nonfundamental G

Consider the possibility that the true fundamental constants of nature are \hbar , c , and l^2 , where the last constant has the dimensions of area. This suggests (as mentioned in Sec. 2 of [6]) that the classical Newton constant G arises from the appropriate *ratio* of the two quantum constants l^2 and \hbar :

$$G = f c^3 l^2 / \hbar, \quad (2)$$

with a positive numerical factor $f \in \mathbb{R}^+$ to be calculated from the microscopic theory. Note that, for $f = 1$, the fundamental length l equals the standard Planck length $l_P \equiv (\hbar G)^{1/2} c^{-3/2}$.

Expression (2) leads to the following structure of the Newtonian gravitational acceleration $A_{\text{grav}} \equiv |\mathbf{A}_{\text{grav}}|$ from a point mass with a macroscopic value M at a macroscopic distance R :

$$A_{\text{grav}} = GM/R^2 = c (f M c^2 / \hbar) (l^2 / R^2), \quad (3)$$

with all microscopic quantities indicated by lower-case symbols. The structure on the right-hand side of (3) is suggestive: the fundamental velocity c is multiplied by a mass-induced decay rate of space, $f M c^2 / \hbar$ with coupling constant f , and a geometric dilution factor, l^2 / R^2 .

Precisely this structure can be seen to result from the reasoning of Verlinde (see, in particular, Sec. 3 of [1]) for a spherical holographic screen Σ_{sph} with area $A = 4\pi R^2$ (Fig. 1):

$$\begin{aligned} A_{\text{grav}} &\stackrel{\textcircled{1}}{=} 2\pi c (k_B T / \hbar) \\ &\stackrel{\textcircled{2}}{=} 4\pi c (f \tfrac{1}{2} N k_B T / \hbar) (f^{-1} / N) \\ &\stackrel{\textcircled{3}}{=} 4\pi c (f E / \hbar) (l^2 / A) \\ &\stackrel{\textcircled{4}}{=} c (f M c^2 / \hbar) (l^2 / R^2), \end{aligned} \quad (4)$$

where step ① relies on the Unruh effect [11], step ② on trivial mathematics, and step ③ on the following relation between the effective number N of degrees of freedom of the holographic screen and the area A of the screen:

$$N = f^{-1} A / l^2. \quad (5)$$

Step ③ of (4) also assumes that the screen corresponds to a physical system in a state of equilibrium (or close to it), with a uniform distribution of the microscopic degrees of freedom over the surface and equipartition of the total energy E over these degrees of freedom (both properties being consistent with having a screen given by a constant-curvature manifold, i.e., a spherical surface). Somewhat surprisingly, Lorentz invariance is seen to play a role in steps ① and ④ of (4): implicitly as the Unruh temperature

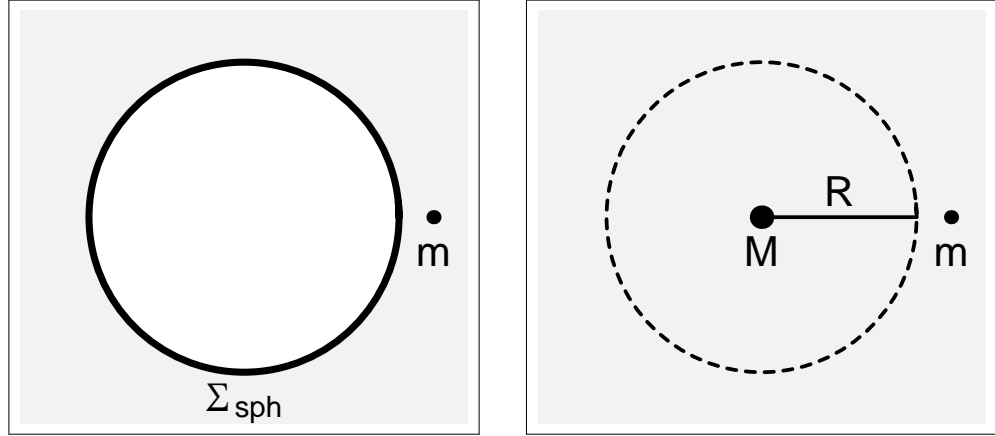


Figure 1. Left panel: Spherical holographic screen Σ_{sph} with area $A = 4\pi R^2$ and test mass m in the emerged space (shaded) outside the screen [1]. The screen Σ_{sph} has N microscopic degrees of freedom at an equilibrium temperature T with total equipartition energy $E = \frac{1}{2} N k_B T$. Right panel: The gravitational effects of Σ_{sph} for the emergent space correspond, in leading order, to those of a point mass $M = E/c^2$ located at the center of a sphere with radius R . (The Schwarzschild radius $R_{\text{Schw}} \equiv 2GM/c^2$ is considered to be negligible compared to R and cannot be shown in the right panel, but the corresponding sphere would be a maximally-coarse-grained screen with smallest possible area, according to [1].)

ultimately traces back to the Lorentz invariance of the Minkowski vacuum [11] and explicitly through the energy-mass equivalence $E \equiv M c^2$ from special relativity.

The several steps of (4) constitute, if confirmed by the definitive microscopic theory, a *derivation* of Newton’s gravitational coupling constant G in the form (2). The point of view of this article is not to consider (4) as mere dimensional analysis but to take all numerical factors seriously. In that spirit, there is the new insight from (5) that, given the “quantum of area” l^2 , the inverse of the constant f entering Newton’s constant (2) is related to the *nature* of the microscopic degrees of freedom on the holographic screen. For example, an “atom of space” with “spin” s_{atom} gives $f^{-1} = 2 s_{\text{atom}} + 1$, but this “spin” need not be half-integer. Still, the number of “atoms” needed to build-up the area A is taken to be an integer, given by the ratio of the area A and the quantum l^2 .

3. Two types of entropy

The introduction of *two* quantum constants, \hbar and l^2 , may also help to resolve a potential problem noted by Verlinde in Sec. 6.4 of [1]. There, he considers an equipotential screen Σ which is not a maximally-coarse-grained surface but is nevertheless assumed to be in thermal equilibrium. He, then, remarks that the required entropy S_Σ appears to contradict Bekenstein’s upper bound [12] on the entropy S_Ξ of a material system Ξ with energy E_Ξ and effective radius R_Ξ ,

$$S_\Xi/k_B < 2\pi \hbar^{-1} E_\Xi R_\Xi/c. \quad (6)$$

With the new fundamental constant l^2 , Verlinde’s expression (6.41) for S_Σ is replaced by

$$S_\Sigma/k_B = \frac{1}{4} f^{-1} l^{-2} \int_\Sigma dA, \quad (7)$$

which generalizes (5). Expressions (6) and (7) involve essentially different physics characterized by, respectively, \hbar and l^2 (see also the discussion of Sec. 2 in [6] for a generalized dimensionless action with $\hbar = 0$ and $l^2 > 0$). This observation would appear to support Verlinde’s suggestion that Bekenstein’s bound may not apply to the holographic screen. Still, the puzzle remains how these two types of entropy combine, as they somehow must do in an appropriate limit.

4. Model calculation of G

For a maximally-coarse-grained spherical surface (horizon) with area A , the entropy (7) reproduces the Bekenstein–Hawking black-hole entropy [9, 10]

$$S_{\text{BH}}/k_B = \frac{1}{4} A/(f l^2) = (1/4) N, \quad (8)$$

where the number N has already been defined by (5).

Now, consider the “atoms of space” mentioned in the last paragraph of Sec. 2. The crucial new equation from (5) is then given by

$$N = d_{\text{atom}} N_{\text{atom}}, \quad (9)$$

with the physical interpretation of l^2 as the quantum of area giving

$$N_{\text{atom}} \equiv A/l^2 \in \mathbb{N}_1 \equiv \{1, 2, 3, \dots\} \quad (10a)$$

and the effective dimension of the internal Hilbert space of a single “atom of space” taking values

$$d_{\text{atom}} \equiv f^{-1} \in \mathbb{R}^+. \quad (10b)$$

The physical picture, suggested by the derivation (4), is that the “atoms of space” have no translational degrees of freedom but only internal degrees of freedom.

The number of configurations [2, 3] of these distinguishable “atoms of space” is readily calculated:

$$N_{\text{config}} = \prod_{n=1}^{N_{\text{atom}}} d_{\text{atom}} = (d_{\text{atom}})^{N_{\text{atom}}}. \quad (11)$$

Equating this number of configurations with the exponential of the Bekenstein–Hawking entropy (8) while using (9) gives the following set of conditions:

$$(d_{\text{atom}})^{N_{\text{atom}}} = \exp[(1/4) d_{\text{atom}} N_{\text{atom}}], \quad (12)$$

for all positive integer values of N_{atom} . Remarkably, this infinite set of conditions reduces to a single transcendental equation for the effective dimension d_{atom} ,

$$\ln d_{\text{atom}} = (1/4) d_{\text{atom}}, \quad (13)$$

which has two solutions:

$$d_{\text{atom}}^{(+)} \approx 8.613\,169\,456, \quad d_{\text{atom}}^{(-)} \approx 1.429\,611\,825, \quad (14)$$

where a 1 ppb numerical precision suffices for the present purpose. §

Given l^2 , there are then two possible values for the gravitational coupling constant (2):

$$G_{\pm} = (d_{\text{atom}}^{(\pm)})^{-1} c^3 l^2 / \hbar. \quad (15)$$

The detailed microscopic theory must tell which of the two values from (14) enters (15). It could, for example, be that the microscopic theory demands $d_{\text{atom}} \geq 2$, selecting the larger value $d_{\text{atom}}^{(+)}$ in (14) and (15).

The experimental value G_N of Newton’s gravitational coupling constant is, of course, already known [13], albeit with a rather large relative uncertainty of 100

§ Condition (13) would not be satisfied for any value of d_{atom} if the factor 1/4 on the right-hand side, which traces back to (8), were replaced by an arbitrary number $g > 1/e$, with $e \approx 2.71828$ the base of the natural logarithm. Note also that (13) rules out $d_{\text{atom}} = 1$, corresponding to $f = 1$ in the original expression (2) for Newton’s constant.

ppm [14]. A more practical interpretation of result (14) is, therefore, to calculate two possible values for the “quantum of area”:

$$(l_{\pm})^2 = d_{\text{atom}}^{(\pm)} (l_P)^2 \approx \begin{cases} 2.2498 \times 10^{-69} \text{ m}^2, \\ 3.7343 \times 10^{-70} \text{ m}^2, \end{cases} \quad (16)$$

with $l_P \equiv (\hbar G_N)^{1/2}/c^{3/2} \approx 1.6162 \times 10^{-35} \text{ m}$ for $G_N = 6.6743(7) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ [14]. The microscopic theory would, again, have to choose between these alternative values. For either choice, the implication would be that l and l_P are of the same order of magnitude.

Needless to say, the numerical estimates of (16) are only indicative because of the extreme simplification of the model calculation (for example, merely “tiles” of a single size l^2 and design d_{atom} have been used to cover the area A). But, perhaps, the simplicity of the model is also its strength, as long as the *effective* quantum of area l^2 is considered and not the individual eigenvalues of the area operator.

The real question is if this l^2 can be measured directly. This question will be addressed in the next section. Anticipating a positive outcome of that discussion and looking far into the future, note that the accurate measurement of one of the values of l^2 in (16) would allow for an equally accurate calculation of G from (15). For example, measuring for l^2 the larger value in (16) with a relative uncertainty of 100 ppb would give G also with an uncertainty of approximately 100 ppb from (15) by use of the $d_{\text{atom}}^{(+)}$ value from (14), since \hbar is already known with an uncertainty of 50 ppb [14].

5. Experiments

As promised in the previous section, let us briefly discuss the prospects of the experimental determination of the factor f in (2), which may or may not be found to agree with the inverse of one of the calculated values in (14). Given the numerical values for c and \hbar from nongravitational experiments, at least *two* gravity/spacetime measurements would be needed to disentangle f and l .

The first measurement is, of course, provided by the Cavendish experiment [13, 14], which determines the particular combination $f l^2$.

A second measurement (without definite results, for the moment) can come from

cosmic-ray particle-propagation experiments probing Lorentz-violating effects^{||} from a nontrivial small-scale structure of spacetime [15, 16]. Such a measurement may, in fact, determine $f^2 \equiv (l_P/l)^4$, if the average size of spacetime defects is set by l_P and their average separation by l (with $l > l_P$); see the discussion of the paragraph starting a few lines below Eq. (10) in [6]. The value $f^2 \gtrsim 10^{-2}$ suggested by (14) would, however, be hard to reconcile with the data (cf. [15, 16] and references therein).

A third type of measurement (entirely in the domain of *Gedankenexperiments*) could try to isolate pure-quantum-gravity effects of (primordial) gravitational waves. Such a measurement would only depend on l^2 , if the generalized dimensionless action of [6] is relevant.

A fourth type of measurement (also in the domain of *Gedankenexperiments*) would look for quantum modifications of Newton's gravitational acceleration (3) by a multiplicative factor $[1 - \tilde{a} l^2/R^2]$, where the dimensionless number \tilde{a} would trace back to a logarithmic correction of the entropy (7), as pointed out in [17]. More generally, an entropy modification $S(A) = (1/4) k_B l_P^{-2} [A + l^2 \tilde{s}(A/l^2)]$, for some dimensionless function \tilde{s} of A/l^2 , would give a correction factor $[1 + l^2 d\tilde{s}/dA]$ for Newton's gravitational force. A measurement of such a modification of the force could, in principle, be used to determine l^2 , if the function $\tilde{s}(A/l^2)$ is nontrivial and known from theory.

Each of the last three possible experiments relies on a crucial assumption (indicated by occurrence of the word 'if') and is, therefore, not yet conclusive in determining the value of l^2 .

6. Conclusion

The two most interesting results of this article are the following. The first is that the interpretation of the Newtonian acceleration (3) as a mass-induced decay rate of space (together with a geometric dilution factor) may be explained by a Verlinde-type derivation (4) relying on the Unruh temperature and holography. The second

^{||} According to the discussion in Sec. 2, it may be that the fundamental theory is essentially Lorentz invariant. Still, there may be effects from some type of spontaneous symmetry breaking of Lorentz invariance (meaning that a particular ground-state solution breaks the symmetry), which show up as modifications of the standard particle-propagation properties.

is the single transcendental equation (13), which allows for an explicit calculation of the numerical factor $f \equiv (d_{\text{atom}})^{-1}$ entering expression (2) for Newton's gravitational constant, where the microscopic theory is still needed to choose between the two possible values (14).

Having a calculated value for f in the G formula (2) is, of course, only of interest if l^2 can be determined directly (the numerical values of G , \hbar , and c are already known). The experiments discussed in Sec. 5 are suggestive but, for the moment, still inconclusive, because each experiment involves one or more assumptions. The main outstanding task, therefore, is to design an experiment, real or imaginary, which allows for an unambiguous determination of the quantum-gravity length scale l , independent of the value of the Planck length l_P [even though, in the end, both may turn out to have approximately the same numerical value, as suggested by the calculated numbers (16)].

The first version of the present article was released on June 10, 2010. Since then, it has been shown [18] that a more sophisticated tiling than the one used in Sec. 4 can produce a single transcendental equation which gives a *unique* physical value for l^2 . The numerical values for l^2 from two such tiling models are both approximately equal to $2.6 \times 10^{-69} \text{ m}^2$, which is only 20 % above the maximal value found here. More importantly, the quantity l^2 of these models [18] would correspond to the true minimal quantum of area.

Acknowledgements

The author thanks D. Easson for pointing out [1] and S. Hellerman, H. Sahlmann, L. Smolin, W. Unruh, and E. Verlinde for discussions. He also gratefully acknowledges the hospitality of the IPMU during the month of May 2010. This work was supported in part by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

References

- [1] E.P. Verlinde, “On the origin of gravity and the laws of Newton,” arXiv:1001.0785v1.
- [2] G. ’t Hooft, “Dimensional reduction in quantum gravity,” in: A. Ali, J. Ellis, and S. Randjbar-Daemi (eds.), *Salamfestschrift: A Collection of Talks*, World Scientific, Singapore, (1993), arXiv:gr-qc/9310026.
- [3] L. Susskind, “The world as a hologram,” J. Math. Phys. **36**, 6377 (1995), arXiv:hep-th/9409089.
- [4] T. Jacobson, “Thermodynamics of space-time: The Einstein equation of state,” Phys. Rev. Lett. **75**, 1260 (1995), arXiv:gr-qc/9504004.
- [5] T. Padmanabhan, “Equipartition of energy in the horizon degrees of freedom and the emergence of gravity,” Mod. Phys. Lett. A **25**, 1129 (2010), arXiv:0912.3165.
- [6] F.R. Klinkhamer, “Fundamental length scale of quantum spacetime foam,” JETP Lett. **86**, 73 (2007), arXiv:gr-qc/0703009.
- [7] A.D. Sakharov, “Vacuum quantum fluctuations in curved space and the theory of gravitation,” Sov. Phys. Dokl. **12**, 1040 (1968); reprinted in: Gen. Rel. Grav. **32**, 365 (2000).
- [8] M. Visser, “Sakharov’s induced gravity: A modern perspective,” Mod. Phys. Lett. A **17**, 977 (2002), arXiv:gr-qc/0204062.
- [9] J.D. Bekenstein, “Black holes and entropy,” Phys. Rev. D **7**, 2333 (1973).
- [10] S.W. Hawking, “Particle creation by black holes,” Commun. Math. Phys. **43**, 199 (1975); Erratum-ibid. **46**, 206 (1976).
- [11] W.G. Unruh, “Notes on black hole evaporation,” Phys. Rev. D **14**, 870 (1976).
- [12] J.D. Bekenstein, “A universal upper bound on the entropy to energy ratio for bounded systems,” Phys. Rev. D **23**, 287 (1981).
- [13] H. Cavendish, “Experiments to determine the density of the Earth,” Phil. Trans. R. Soc. Lond. **88**, 469 (1798) [doi:10.1098/rstl.1798.0022].
- [14] P.J. Mohr, B.N. Taylor, and D.B. Newell, “CODATA recommended values of the fundamental physical constants: 2006,” Rev. Mod. Phys. **80**, 633 (2008), arXiv:0801.0028.
- [15] S. Bernadotte and F.R. Klinkhamer, “Bounds on length scales of classical spacetime foam models,” Phys. Rev. D **75**, 024028 (2007), arXiv:hep-ph/0610216.
- [16] F.R. Klinkhamer, M. Schreck, “New two-sided bound on the isotropic Lorentz-violating parameter of modified Maxwell theory,” Phys. Rev. D **78**, 085026 (2008), arXiv:0809.3217.
- [17] L. Modesto and A. Randono, “Entropic corrections to Newton’s law,” arXiv:1003.1998v1.
- [18] H. Sahlmann, “Newton’s constant from a minimal length: Additional models,” Class. Quantum Grav. **28**, 015006 (2011), arXiv:1010.2650.